

Aggregate planning

-Aggregate planning :

is concerned with determining the quantity and timing of production for the intermediate future, often from 3 to 18 months.

هي عملية تحديد حجم وميعاد الإنتاج لفترة زمنية متوسطة تتراوح بين ٣ إلى ١٨ شهر

-Operation managers try to determine the best way to meet forecasted demand by adjusting:

العوامل التي تعتمد عليها تلبية الطلبات من الإنتاج

- 1- production rates
- 2- labor levels
- 3- Inventory levels
- 4- Overtime work
- 5- Subcontracting rates

- Objective of aggregate planning is to meet forecasted demand while minimizing cost over the planning period

- Costs :

1- Production Costs :

- material
- Labor
- over time
- Sub Contracting

2- Inventory Costs :

- holding
- Shortage (back order)

3- Capacity change

- hiring and training workers
تأجير عمال لزيادة الإنتاج
- lays off workers (Firing)
خروج عمال عن ألقاعن الطلب وذلك لعدم حاجة المصنع لهم

Spreadsheet methods

1_ Zero inventory plan (lot for lot)

produce exactly the demand for each period, which requires a fluctuating work force, no inventory is carried

يتم إنتاج المطلوب من كل فترة بدون زيادة وذلك يتطلب تغيير عدد العمال
أعتماداً على حجم الإنتاج المطلوب، مما يترتب عنه عدم وجود منتجات في المخزون

Workers are added when demand increases and are level off when demand decreases.

Example 2-1:

X-Print Manufacturing makes laser printers. One plant assembles the PL-4000 model. Standards indicate that one worker can assemble five printers per day. This model costs

about \$350 to make, and the company figures it costs \$5 to hold one printer in inventory for one month. Workers earn \$1500 per month and can be hired for \$500 each; firing a worker costs \$750. Currently, there are 12 workers in the assembly department. If a printer is backordered, the cost is \$35 per unit per month.

Month	July	Aug	Sep	Oct	Nov	Dec	total
working days	21	22	21	23	19	20	126
Demand	1020	950	800	1000	1250	650	5670

Given:

1. Capacity (worker productivity)

production = 5 unit / worker / day

2. Costs:

$$\text{making Cost} = \$350 / u$$

$$\text{holding Cost} = \$5 / u / \text{month}$$

$$\text{المرتب} \\ \text{Wages} = \$1500 / \text{worker} / \text{month}$$

$$\text{hiring Cost} = \$500 / \text{worker}$$

$$\text{firing Cost} = \$750 / \text{worker}$$

$$\text{Shortage Cost} = \$35 / u / \text{month} \\ (\text{back order})$$

3. Number of existing labours :

Currently 12 workers

Solution:

		Jul	Aug	Sep	Oct	Nov	Dec	Total
1	Days	21	22	21	23	19	20	126
2	u/worker	105	110	105	115	95	100	630
3	Demand	1020	950	800	1000	1250	650	5670
4	Workers needed	10	9	8	9	14	7	57
5	" available	12	→ 10	→ 9	→ 8	→ 9	→ 14	-
6	" hired	0	0	0	1	5	0	6
7	Hiring Cost	0	0	0	500	2500	0	3000
8	Worker laid off	2	1	1	0	0	7	11
9	lay off Cost	1500	750	750	0	0	5250	8250
10	Workers used	10	9	8	9	14	7	57
11	labor Cost	15000	13500	12000	13500	21000	10500	85500
12	units produced	1020	950	800	1000	1250	650	5670
13	Net inventory	0	0	0	0	0	0	0
14	Holding Cost	0	0	0	0	0	0	0
15	Backorder Cost	0	0	0	0	0	0	0
16	Total Cost	16500	14250	12750	14000	23500	15750	96750

$$\begin{aligned} \text{making Cost} &= 350 * \text{total units produced} \\ &= 350 * 5670 = 1,984,500 \end{aligned}$$

$$\text{Total Cost} = 1,984,500 + 96750 = 2,081,250$$

Rows description:

1- No. of days / month : given

2- No. of units produced by a worker per month :

$$\textcircled{2} = \textcircled{1} * \text{unit/worker/day}$$

3- Demand : given

$$4- \text{Workers needed} = \frac{\text{Demand}}{\text{u/worker}}$$

$$\textcircled{4} = \frac{\textcircled{3}}{\textcircled{2}} \quad \text{تقريب للأعلى}$$

5- Workers available :

- put the no. of existing workers (given) in the first month

أكتب عدد العمال الموجود من المصنع (المعطى) في أول شهر

- and write down the no. of row $\textcircled{4}$

وبعد ذلك يتم كتابة أعداد العمال من المصنع 4

6 & 8 -

$$\text{Workers available} - \text{needed} = \begin{cases} + \text{Firing} \\ - \text{hiring} \end{cases}$$

$$\textcircled{5} - \textcircled{4} = \begin{cases} \rightarrow + \textcircled{8} \\ \rightarrow - \textcircled{6} \end{cases}$$

7 & 9 -

$$\textcircled{7} = \textcircled{6} * \text{hiring Cost}$$

$$\textcircled{9} = \textcircled{8} * \text{Firing Cost}$$

$$10 - \text{Workers used} = \text{workers needed}$$

$$\textcircled{10} = \textcircled{4}$$

$$\begin{aligned} 11 - \text{Labor Cost} &= \text{Workers used} * \text{Wages} \\ &= \textcircled{10} * \text{Wages} \end{aligned}$$

$$\begin{aligned} 12 - \text{Units produced} &= \text{Workers used} * \text{u/worker} \\ &= \textcircled{10} * \textcircled{2} \\ \textcircled{00} &= \text{Demand} = \textcircled{3} \end{aligned}$$

13 & 14 & 15 - Zeros (Zero inventory plan)

2- Level work force plan:

level work force plan or constant work force plan where the same number of workers is used in each period

2.1 Constant production: with backorders

$$\begin{aligned} \text{no. of workers} &= \frac{\sum \text{demand}}{\sum \text{capacity}} = \\ &= \frac{1020 + 950 + \dots + 650}{105 + 110 + \dots + 100} = \frac{5670}{630} = 9 \end{aligned}$$

av. no. of workers
↓

2.2 Constant production: with no backorders

	Jul	Aug	Sep	Oct	Nov	Dec
① w/worker	105	110	105	115	95	100
② = Cum ①	105	105+110=215	320	435	530	630
③ Demand	1020	950	800	1000	1250	650
④ = Cum ③	1020	1970	2770	3770	5020	567
⑤ = $\frac{④}{②}$	9.7 ≈ 10	10	9	9	10	1

Select max from ⑤ → no. of workers = 10

Solution of example 2.1

with back orders

		Jul	Aug	Sep	Oct	Nov	Dec	Total
1	Days	21	22	21	23	19	20	126
2	u/worker	105	110	105	115	95	100	630
3	Demand	1020	950	800	1000	1250	650	5670
4	workers needed	9	9	9	9	9	9	54
5	" available	12	9	9	9	9	9	-
6	hired	0	0	0	0	0	0	0
7	hiring cost	0	0	0	0	0	0	0
8	worker layoff	3	0	0	0	0	0	0
9	layoff cost ⑧ * 750	2250	0	0	0	0	0	2250
10	workers used ④	9	9	9	9	9	9	54
11	labor cost ⑩ * 1500	13500	13500	13500	13500	13500	13500	81000
12	units produced ⑩ * ②	945	990	945	1035	855	900	5670
13	Net inventory ⑫ - ③ + ⑬	-75	-35	110	145	-250	0	-
14	Holding Cost			550	725			1275
15	Backorder cost	2625	1225			8750		12600
16	Total cost	18375	14725	14050	14225	22250	13500	97125

making Cost = 1,984,500

Total Cost = 1,984,500 + 97125 = 2,081,625

Solution of example 2.1

with no back orders

		Jul	Aug	Sep	Oct	Nov	Dec	Total
1	Days	21	22	21	23	19	20	126
2	U/worker	105	110	105	115	95	100	630
3	Demand	1020	950	800	1000	1250	650	5670
4	workers needed	10	10	10	10	10	10	60
5	„available	12	10	10	10	10	10	-
6	„hired	0	0	0	0	0	0	0
7	hiring cost	0	0	0	0	0	0	0
8	workers laidoff	2	0	0	0	0	0	0
9	layoff cost	1500	0	0	0	0	0	1500
10	workers used	10	10	10	10	10	10	60
11	labor cost	15000	15000	15000	15000	15000	15000	90000
12	units produced	1050	1100	1050	1150	950	¹⁰⁰⁰⁻⁶³⁰ 1000	6300
13	Net inventory	30	180	430	580	280	630 ⁰	-
14	Holding cost	150	900	2150	2900	1400	3150 ⁰	10650
15	Backorder cost	0	0	0	0	0	0	0
16	Total Cost	16650	15900	17150	17900	16400	18150	102150

$$\text{making Cost} = \sum (12) (6300) (350) = 2,205,000$$

$$\text{Total Cost} = 2,307,150$$

Example 2-2:

Poseidon Meter Inc. Makes a variety of water meters. Data for the past year indicate a worker can make, on average, 100 meter per four week period. The inventory holding cost is computed to be \$1 / meter / period. Backorders, if allowed, cost about \$ 2 / meter / period. New workers can be hired at a cost of \$ 1000 / worker, and existing workers can be laid off at a cost of \$ 2000 / worker. Workers are paid \$ 1500 / period. There are currently 10 workers at Poseidon. The Forecast for the next four periods is 1200, 1200, 1000 and 1000 meters respectively.

Given:

1- Capacity : 100 meter/period

2- Costs:

holding Cost = \$ 1 /meter/period

backorder Cost = \$ 2 /meter/period

hiring Cost = \$ 1000 /worker

laid off Cost = \$ 2000 /worker

wage = \$ 1500 /period/worker

3- No. of existing labors:

currently 10 workers

1. Zero Inventory

		1	2	3	4	total
1	Days					
2	u/worker	100	100	100	100	400
3	Demand	1200	1200	1000	1000	4400
4	workers needed	12	12	10	10	44
5	" available	10	12	12	10	-
6	" hired	2	0	0	0	2
7	hiring Cost	2000	0	0	0	2000
8	workers land off	0	0	2	0	2
9	lay off Cost	0	0	4000	0	4000
10	workers used	12	12	10	10	44
11	labor Cost	18000	18000	15000	15000	66000
12	units produced	1200	1200	1000	1000	4400
13	net inventory	0	0	0	0	0
14	Holding Cost	0	0	0	0	0
15	Backorder cost	0	0	0	0	0
16	Total Cost	20000	18000	19000	15000	72000

2-level workforce (with backorders)

$$\text{no. of workers} = \frac{\sum \text{Demand}}{\sum \text{Capacity}} = \frac{4400}{400} = 11$$

		1	2	3	4	Total
1	days					
2	u/worker	100	100	100	100	400
3	Demand	1200	1200	1000	1000	4400
4	workers needed	11	11	11	11	44
5	" available	10	11	11	11	-
6	" hired	1	0	0	0	1
7	hiring cost	1000	0	0	0	1000
8	workers laidoff	0	0	0	0	0
9	laidoff cost	0	0	0	0	0
10	workers used	11	11	11	11	44
11	labor cost	16500	16500	16500	16500	66000
12	units produced	1100	1100	1100	1100	4400
13	net inventory	-100	-200	-100	0	-
14	holding cost	0	0	0	0	0
15	backorder cost	200	400	200	0	800
16	Total	17700	16900	16700	16500	67800

3. level workforce : (no. backorders)

Cum demand	1200	2400	3400	4400
Cum Capacity	100	200	300	400
no. of workers	12	12	12	11

		1	2	3	4	total
1	days					
2	worker	100	100	100	100	400
3	demand	1200	1200	1000	1000	4400
4	workers needed	12	12	12	12	48
5	" available	10	12	12	12	-
6	" hired	2	0	0	0	2
7	hiring cost	2000	0	0	0	2000
8	workers laid off	0	0	0	0	0
9	laid off cost	0	0	0	0	0
10	workers used	12	12	12	12 8	40 48
11	labor cost	18000	18000	18000	18000	72000
12	units produced	1200	1200	1200	1200 ⁻⁴⁰⁰ 800	
13	net inventory	0	0	200	400 0	200
14	holding cost	0	0	200	0	200
15	backorder cost	0	0	0	0	0
16	total cost	20000	18000	18200	18000	74200

* Linear programming approaches

①

to aggregate planning;

Objective Function:

$$\begin{aligned} \text{Minimize } & \sum_{t=1}^T \text{Costs} \\ & \text{making wages} \quad \text{hiring \& laying off} \\ & \sum \left[C_t^P P_t \right] + \left[C_t^W W_t \right] + \left[C_t^H H_t + C_t^L L_t \right] \\ & + \left[C_t^I I_t + C_t^B B_t \right] \\ & \text{inventory \& backorder} \end{aligned}$$

where:

$\{ P_t : \# \text{ of units produced in period } t$
 $\{ C_t^P : \text{Cost to produce one unit in period } t$

$\{ W_t : \# \text{ of workers available}$
 $\{ C_t^W : \text{Cost of one worker}$

$\{ H_t : \# \text{ of workers hired}$
 $\{ C_t^H : \text{Cost to hire one worker}$
 $\{ L_t : \# \text{ of workers laid off}$
 $\{ C_t^L : \text{Cost to lay off one worker}$

$$\left\{ \begin{array}{l} I_t : \# \text{ of units held in inventory at the end of period } t \\ C_t^I : \text{Cost to hold one unit in inventory for period } t \\ B_t : \# \text{ of units back ordered at the end of period } t \\ C_t^B : \text{Cost to back order one unit for period } t \end{array} \right. \quad (2)$$

Subject to

Production

1. Capacity Constraints

$$P_t \leq n_t W_t \quad t = 1, 2, \dots, T$$

n_t : worker productivity in period t

2. Work Force Constraints

$$W_t = W_{t-1} + H_t - L_t \quad t = 1, 2, \dots, T$$

inventory balance

3. ~~Material~~ Constraints (Inventory)

net inventory this period = net inventory last period + production
this period - demand this period

$$[I_t - B_t] = [I_{t-1} - B_{t-1}] + P_t - D_t$$

$$P_t, W_t, H_t, L_t, I_t \geq 0$$